

Problem Set #10

Exercise 1: (★) 4 points

Show that if $p \geq 7$ is an odd prime, then

$$\left(\frac{n}{p}\right) = \left(\frac{n+1}{p}\right)$$

for at least one of $n = 2, 4, 5$, or 8 .

Exercise 2: (★) 4 points

The prime p for which $x^2 \equiv 13 \pmod{p}$ has solutions.

Exercise 3: (★★) 4 points

Compute the continued fraction expansions of $53/18$ and $115/53$.

Exercise 4: (★★★) 4 points

If $x = [a_0, \dots, a_n, b]$ and $y = [a_0, \dots, a_n, c]$ with $b < c$, then $x < y$ if n is odd and $x > y$ if n is even.

Exercise 5: (★★★) 4 points

Compute the continued fraction expansion of $\sqrt{17}$ and $\sqrt{19}$ (stop when you find the period), use this to find their first five convergents.

¹(★) = easy , (★★)= medium, (★★★)= challenge